the right member of the proposed identity. Hence

$$1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\frac{1}{11}-\cdots=\sqrt{2}\left(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots\right).$$

III. SOLUTION BY J. W. CAMPBELL, University of Chicago.

Let S = sum of the series,

$$\sin 2\theta-\frac{1}{2}\sin 4\theta+\frac{1}{3}\sin 6\theta-\cdots.$$

Then S is the coefficient of i in $e^{2i\theta}-\frac{1}{2}e^{4i\theta}+\frac{1}{3}e^{6i\theta}-\cdots$, that is, in $\log_e(e^{2i\theta}+1)$, or in $\log_e e^{i\theta}(e^{i\theta}+e^{-i\theta})$, or finally in $i\theta+\log_e(2\cos\theta)$, which is θ itself.

Take $\theta=\pi/8$. Then

$$\frac{\pi}{8}=\frac{1}{\sqrt{2}}-\frac{1}{2}+\frac{1}{3\sqrt{2}}+0-\frac{1}{5\sqrt{2}}+\frac{1}{6}\cdots$$

$$=\frac{1}{\sqrt{2}}\left(1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}\cdots\right)-\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{5}\cdots\right).$$

That is,

$$1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\cdots=\sqrt{2}\left[\frac{\pi}{8}+\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{5}\cdots\right)\right]. \tag{1}$$

By Gregory's Series (Loney's *Trigonometry*, Part II, § 94),

$$\theta=\tan\theta-\frac{1}{3}\tan^3\theta+\frac{1}{5}\tan^5\theta-\cdots,\quad\left(-\frac{\pi}{4}\leqq\theta\leqq\frac{\pi}{4}\right).$$

Take $\theta=\pi/4$. Then $\pi/4=(1-1/3+1/5-\cdots)$. Hence,

$$\frac{\pi}{8}=\frac{1}{2}\left(1-\frac{1}{3}+\frac{1}{5}-\cdots\right). \tag{2}$$

Therefore, from (1) and (2),

$$1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\cdots=\sqrt{2}\left(1-\frac{1}{3}+\frac{1}{5}-\cdots\right).$$

Also solved by C. E. HORNE, A. L. MCCARTY, A. M. HARDING, DAVID F. KELLEY, ELMER SCHUYLER, C. N. SCHMALL, and J. W. CLAWSON.

ALGEBRA.

397. Proposed by W. H. BUSSEY, University of Minnesota.

12 oxen are turned into a pasture of $3\frac{1}{3}$ acres and eat all the grass in 4 weeks so that the pasture is bare. 21 oxen are turned into a pasture of 10 acres and eat all the grass in 9 weeks. How many oxen will eat all the grass of 24 acres in just exactly 18 weeks, it being assumed that the grass in all the pastures is at the same height when the oxen are turned in, and that the grass grows at a uniform rate.

I. SOLUTION BY CHRISTIAN HORNUNG, Tiffin, Ohio.

Let x = number of pounds of grass each ox eats per week,
y = number of pounds of grass on each acre at first,
z = number of pounds of grass that grows per week per acre,
k = the number of oxen required in the last condition.

Then $\frac{10}{3}y + \frac{40}{3}z = 48x$, $10y + 90z = 189x$, and $24y + 432z = 18kx$. Eliminating y, we find $10z = 9x$, and $560z = (30k - 576)x$, from which k is found to be 36, the required number of oxen.

II. SOLUTION BY ARTEMAS MARTIN, Washington, D. C.

In the first case one ox eats 1/4 of $3\frac{1}{3}/12$ or 5/72 of an acre, and 5/18 of the growth of that acre, in one week. In the second case one ox eats 1/9 of 10/21 or 10/189 of an acre, and 10/21 of what grows on an acre, in one week.

Since one ox eats the same quantity of grass in one week in each case, therefore, $10/21 - 5/18 = 25/126$ of the growth of one acre during one week is $5/72 - 10/189 = 25/1512$ of an acre; and $25/1512 \div 25/126 = 1/12$ of an acre, what grows on an acre during one week.

$5/72 + 5/18$ of $1/12 = 5/54$, the part of the original quantity of grass on one acre which one ox eats in one week.

$5/54 \times 18 = 5/3$, the quantity of grass, in acres, one ox will eat in 18 weeks.

$24 + (1/12 \times 24 \times 18) = 60$, the quantity of grass, in acres, to be eaten from 24 acres in 18 weeks; and $60 \div 5/3 = 36$, the number of oxen required to eat it.

For other solutions, see my paper on "The 'Pasturage Problem,'" published in the *Mathematical Magazine*, Vol. I, No. 2 (April, 1882), pp. 17–22; also, No. 3 of same volume, pp. 43–44.

Also solved by ALBERT N. NAUER, M. E. GRABER, G. W. HARTWELL, H. C. FEEMSTER, DANIEL KRETH, ELMER SCHUYLER, HORACE OLSON, S. W. REAVES, P. PEÑALVER, and J. W. CLAWSON.

GEOMETRY.

425. Proposed by V. M. SPUNAR, Chicago, Illinois.

Find the ratio of the areas A_1 and A_2 of the parabolas formed by projectiles whose ranges are the same and whose angles of projection are complements of each other.

SOLUTION BY H. C. FEEMSTER, York College, York, Nebraska.

Let $gx^2 - 2v^2x \cos\theta_1 \sin\theta_1 + 2yv^2 \cos^2\theta_1 = 0$ and $gx^2 - 2v^2x\cos\theta_2\sin\theta_2 + 2yv^2\cos^2\theta_2 = 0$ be the two required parabolas, where $\theta_1 + \theta_2 = 90°$, and v is the number of feet per second in the initial velocity. Then

$$A = \int_0^{\frac{2v^2 \sin\theta\cos\theta}{g}} \frac{2v^2x\cos\theta\sin\theta - gx^2}{2v^2\cos^2\theta}\,dx = \frac{2}{3}\,\frac{v^4\sin^3\theta\cos\theta}{g^2},$$